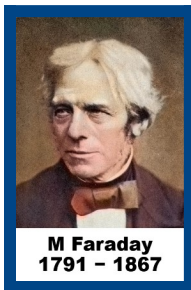
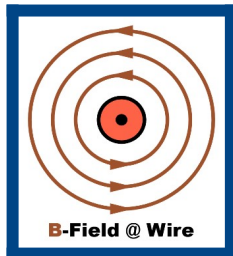


DC Electric Motors & Wire Coils EM Color



By 1820, scientists noticed that a compass needle aligned circumferentially around a direct current (DC) in a conductor. Current flow direction sets the compass “North” orientation.

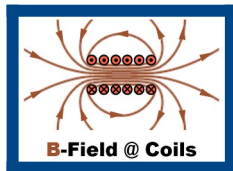
Michael Faraday envisioned magnetic (**B**-field) lines surrounding a DC wire. In Maxwell's Equations, a magnetostatic 4th Law gives a circular **B**-field. If a wire with DC current (*I*) is wrapped around an iron core (n_c) times, a bar magnet exists.



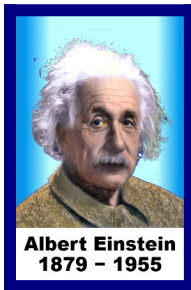
4th Law: $\oint (\mathbf{B} \cdot d\ell) = n_c \mu_0 \iint (\mathbf{J} \cdot \hat{n}_s) ds$ & $I = \iint (\mathbf{J} \cdot \hat{n}_s) ds$
 $\oint (\mathbf{B}_r \cdot d\ell) = |\mathbf{B}_r| (2 \pi r) = n_c \mu_0 I \quad \diamond \quad |\mathbf{B}_r| = (n_c \mu_0 I) / (2 \pi r)$

The 3rd Law gives impeding Voltage (*V*) from change in applied magnetic flux ($\Delta\Phi_B$) through magnet coils. The flux increases due to bar magnet alignment.

3rd Law: $\oint (\mathbf{E} \cdot d\ell) = -d/dt \iint (\mathbf{B} \cdot \hat{n}_s) ds$
 $V = n_c \oint (\mathbf{E} \cdot d\ell) \quad \& \quad \Phi_B \equiv \iint (\mathbf{B} \cdot \hat{n}_s) ds \quad \diamond \quad V = -n_c \Delta\Phi_B / \Delta t$



Einstein Extremes



Since **1905**, **Einstein** has made revolutionary **advances** in Physics. He refined **Quantum ideas** & invented **Relativity**.

From his **photon theory**, light is transmitted in "quanta" of **Energy** (E_0):

$$E_0(\nu) = h\nu$$

From **Newton** in 1687, 3-vector lengths ($r^2 \equiv \mathbf{r} \cdot \mathbf{r}$)

are constant in spatial **rotations**. Mass (Δm_0) & Time (Δt) are **scalars**.

3D Momentum: $\mathbf{p} \equiv m_0 \mathbf{u}$

Kinetic Energy: $E = \frac{1}{2} m_0 u^2$

With **Special Relativity**, we live in 4D **spacetime**, clock rates vary. 4-vector length ($R^\mu R_\mu \equiv (ct)^2 - r^2$) is conserved in spatial rotation, velocity **boost**.

Lorentz Factor: $\gamma \equiv 1/\sqrt{1-u^2/c^2}$ with 3D Velocity (\mathbf{u})

3D Momentum: $\mathbf{p} \equiv \gamma m_0 \mathbf{u}$

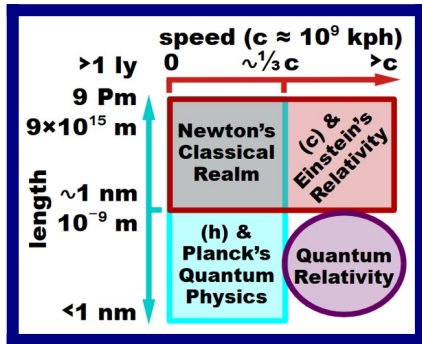
Total Energy: $E \equiv \gamma m_0 c^2 \approx m_0 c^2 + \frac{1}{2} m_0 u^2 + \dots$

4-Position: $R^\mu \equiv (ct, \mathbf{r})$

4-Momentum: $P^\mu \equiv (E/c, \mathbf{p})$

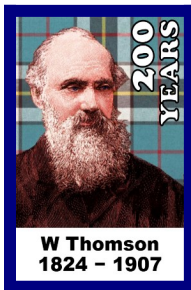
Einstein helped **define extremes** setting challenges in **Modern Physics**.

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Finding Absolute Zero



From the thermometer invented in 1714, ideal gas behavior was given in Charles's Law by 1787:

$$T_1/V_1 = T_2/V_2$$

The Ideal Gas Law approximates certain gases @ molecular mass (m), temperature (T_i), volume (V_i) w/restrictions:

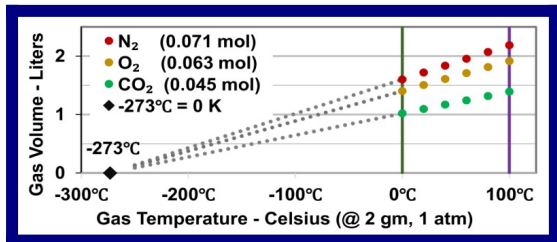
a) average Kinetic Energy $\equiv \langle \frac{1}{2}mv^2 \rangle \propto T$.

b) point particles w/no interactions.

c) constant motion w/elastic collisions.

In 1848, William Thomson (Lord Kelvin) proposed an Absolute Zero (0 K) @ -273°C extrapolated from near ideal gas data between 0°C & 100°C .

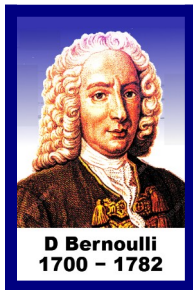
local	add \diamond \diamond subtract	absolute
Celsius ($^\circ\text{C}$)	± 273.15	Kelvin (K)
Fahrenheit ($^\circ\text{F}$)	± 459.67	Rankine (R)



At Absolute Zero, particle thermal kinetic energy stops. A "Zero-Point Energy" of random quantum motion remains:

$$\Delta x_i \Delta p_i = \frac{1}{2} \hbar \quad \& \quad \Delta E \Delta t = \frac{1}{2} \hbar$$

Ideal Gas Law



In 1738, **Daniel Bernoulli**, published his **kinetic theory** of gases. Molecules exert pressure through elastic collisions with container walls. For a gas @ absolute **temperature** (T) w/particles having molecular **mass** (m), **speed** (v), **kinetic energy** ($KE \equiv \frac{1}{2}mv^2$), the theory requires:

- Boltzmann constant** (k_B) relates $\langle KE \rangle \propto \langle mv^2 \rangle = k_B T$.
- constant motion w/elastic collisions.**
- point particles w/o interactions.**

Pressure (p_A) on container wall w/area (A) is **force** (F) per area of count (N) particles w/reversed velocity ($2mv_x$). An average is taken over **time** (Δt).

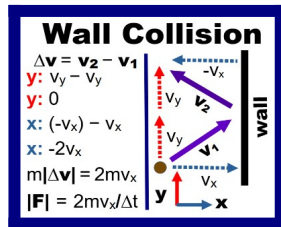
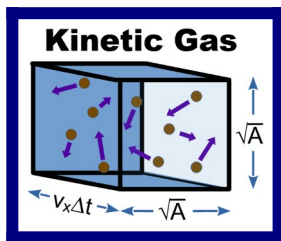
$\frac{1}{2}N$ particles collide w/wall A in time Δt ($v_x > 0$)

Container has **volume** (V): $V = Av_x \Delta t \Leftrightarrow 1/A = v_x \Delta t / V$

$$p_A = \frac{1}{2}N \langle |F|/A \rangle = \frac{1}{2}N \langle (2mv/\Delta t) \cdot (v\Delta t/V) \rangle = N \langle mv^2 \rangle / V$$

Ideal Gas Law: $N \langle mv^2 \rangle = N k_B T \Leftrightarrow p_A V = N k_B T$

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Lorentz Force Equation

EM
Color



In 1895, **Hendrik Lorentz** added the **E-field** to **Oliver Heaviside's** magnetic force to state his final **electromagnetic (EM) force equation**:

$$\mathbf{F}_t = q_t [\mathbf{E}(\mathbf{x}_t) + \mathbf{v}_t \times \mathbf{B}(\mathbf{x}_t)]$$

\mathbf{F}_t – 3D vector force (N – Newtons)

\mathbf{x}_t – 3D vector location (m – meters)

\mathbf{v}_t – 3D vector velocity (m/s – meter/second)

q_t – scalar charge (C – **Coulomb**) ($1 \text{ C} \approx -6.2 \times 10^{18} \text{ e}^-$)

$\mathbf{E}(\mathbf{x}_t)$ – 3D **electric field** vector (N/C –

Volts/meter). **E-fields** define EM forces assuming near infinite propagation speeds & static states.

E-fields undergo **work**, store & expend **energy**.

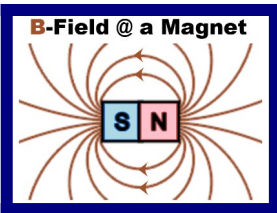
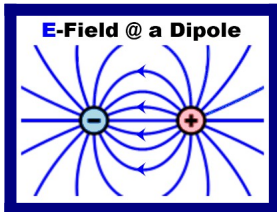
$\mathbf{B}(\mathbf{x}_t)$ – 3D **magnetic field** vector (T – Tesla). **B-fields** set corrections due to a finite **EM propagation speed** ($c \approx 10^9 \text{ ft/s} \approx 10^9 \text{ kph}$). **B-fields** do **NO** work (W_B).

$$\mathbf{F}_B \equiv q_t(\mathbf{v}_t \times \mathbf{B}) \quad \Rightarrow \quad \mathbf{F}_B \perp \mathbf{v}_t \quad \& \quad \mathbf{F}_B \perp \mathbf{B}$$

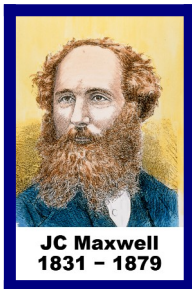
$$d\ell_t \approx \mathbf{v}_t \Delta t \quad \Rightarrow \quad W_B \equiv (\mathbf{F}_B \cdot d\ell_t) = (\mathbf{F}_B \cdot \mathbf{v}_t) \Delta t = q_t [(\mathbf{v}_t \times \mathbf{B}) \cdot \mathbf{v}_t] \Delta t = 0 \quad \Rightarrow \quad W_B = 0$$

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Maxwell's Equations EM Color



By 1862, **James Maxwell** published his **equations** that codified known data of classical **Electromagnetism**.
By 1891, work of **Oliver Heaviside** was credited with their final form.

$$1^{st}) \nabla \cdot \mathbf{D} = \rho$$

$$2^{nd}) \nabla \cdot \mathbf{B} = 0$$

$$3^{rd}) \nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$4^{th}) \nabla \times \mathbf{H} = \mathbf{J} + \partial \mathbf{D} / \partial t$$

$$\oiint (\mathbf{D} \cdot \hat{\mathbf{n}}_s) ds = \iiint \rho dv$$

$$\oiint (\mathbf{B} \cdot \hat{\mathbf{n}}_s) ds = 0$$

$$\oint (\mathbf{E} \cdot d\ell) = -d/dt [\oiint (\mathbf{B} \cdot \hat{\mathbf{n}}_s) ds]$$

$$\oint (\mathbf{H} \cdot d\ell) = \iint (\mathbf{J} \cdot \hat{\mathbf{n}}_s) ds + d/dt [\oiint (\mathbf{D} \cdot \hat{\mathbf{n}}_s) ds]$$

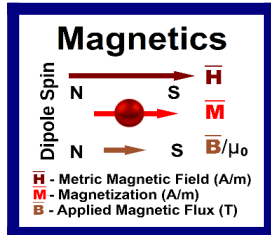
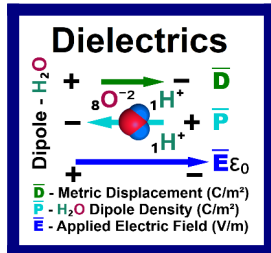
These **equations** were developed in terrestrial settings far from the pure **vacuum** of outer space. In a vacuum $\hookrightarrow \kappa = 0 \hookrightarrow \mathbf{D} = \epsilon_0 \mathbf{E}$ & $\mathbf{H} = \mathbf{B} / \mu_0$. For a gas $|\kappa| < 1$:

$$\mathbf{D} \equiv \epsilon \mathbf{E} \quad \& \quad \epsilon / \epsilon_0 \equiv (1 - \kappa) \quad \hookrightarrow \quad \mathbf{P} = -\kappa \epsilon_0 \mathbf{E} \quad \& \quad \epsilon_0 \mathbf{E} = \mathbf{D} - \mathbf{P}$$

$$\mathbf{H} \equiv \mathbf{B} / \mu \quad \& \quad \mu_0 / \mu \equiv (1 + \kappa) \quad \hookrightarrow \quad \mathbf{M} = \kappa / \mu_0 \mathbf{B} \quad \& \quad \mathbf{B} / \mu_0 = \mathbf{H} - \mathbf{M}$$

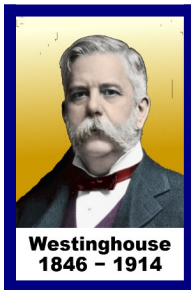
To measure **Electric (E)** & **Magnetic (B)** fields in 1800's labs, **1st order linear Dielectric (P)** & **Magnetic (M)** dipoles were included to match lab results.

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Moving Electricity Efficiently



Westinghouse
1846 - 1914

In 1896, **Westinghouse Electric** gave power to Buffalo, New York 26 miles (42 km) from Niagara Falls. Based on **Ohm's Law**, the design cut heat loss ($P_{Loss} \downarrow$) by reducing the transmission current ($I_{Tran} \downarrow$) across **power line** resistance ($R_{Tran} \bullet$).

Based on **Ohm's Law**, the design cut heat loss ($P_{Loss} \downarrow$) by reducing the transmission current ($I_{Tran} \downarrow$) across **power line** resistance ($R_{Tran} \bullet$).

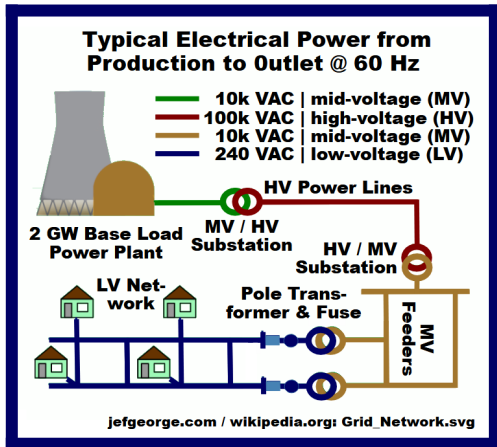
$$\eta_{eff} \equiv (P_{Load} \bullet) / [(P_{Load} \bullet) + (P_{Loss} \downarrow)]$$

$$(P_{Loss} \downarrow) = (R_{Tran} \bullet)(I_{Tran} \downarrow)^2$$

$$(P_{Load} \bullet) = (V_{Tran} \uparrow)(I_{Tran} \downarrow) \eta_{eff}$$

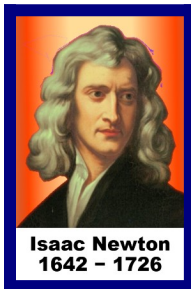
For a set power ($P_{Load} \bullet$), the reduced current ($I_{Tran} \downarrow$) cut losses ($P_{Loss} \downarrow$), driving efficiency (η_{eff}) toward unity. The transmission voltage ($V_{Tran} \uparrow$) was **stepped up** by **transformers** to meet the required load ($P_{Load} \bullet$).

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Newton's Laws of Motion



In 1687, **Isaac Newton** published *Principia* in **Latin**, because most great tomes were from **Roman Antiquity** over 1000 years before. His book described **gravity & 3 Laws of Motion**:

1) "Every object moves in a **straight line** unless acted upon by a force."

2) For a body with **scalars mass** (m) & **acceleration** (a), **force** (F) is:

$F = ma$. The scalar **magnitude** of 3D **vector** (a) is: $a = |a| \equiv (a \cdot a)^{1/2} \Rightarrow F = ma$

3) "For every action, there is an equal & opposite reaction."

Newton's Laws adapted for **Center-of-Mass Analyses**: $\sum F_i = ma$

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Example: Sliding Block

N - normal force
a - acceleration
g - gravity
m - block mass
 μ - friction factor
 φ - incline angle

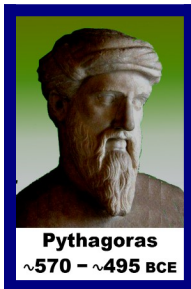
Free Body Diagram

$y: m a_y = N - mg \cos \varphi = 0$
 $y: N = mg \cos \varphi$
 $x: m a_x = mg \sin \varphi - \mu N$
 $x: a_x = g (\sin \varphi - \mu \cos \varphi)$
 $x: a_x = g \cos \varphi (\tan \varphi - \mu)$

$\sum F_i = ma$ Block Slides @ $\varphi > \text{Atan}(\mu)$

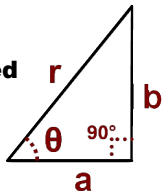
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Pythagorean Theorem



Pythagoras
~570 - ~495 BCE

Around 530 BCE, **Pythagoras** of **Samos** founded his **ascetic** school in **Kroton**, Italy. Its **ideas** influenced **Aristotle**, **Plato** & Renaissance **thinkers**. By 850 CE, **Trigonometry** (Trig) **Functions** used the **theorem** named for him.



$$a^2 + b^2 = r^2 \quad \Leftrightarrow \quad \cos^2\theta + \sin^2\theta = 1$$

with:

$$\cos\theta \equiv a/r \qquad \sin\theta \equiv b/r$$

$$\cos\theta = \pm\sqrt{1 - \sin^2\theta} \qquad \sin\theta = \pm\sqrt{1 - \cos^2\theta}$$



Acute Triangle

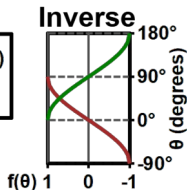
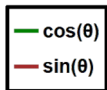
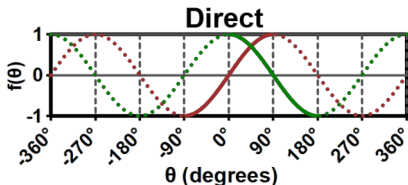


Right Triangle



Obtuse Triangle

The theorem only applies to **right triangles**. “The **hypotenuse** (r) ... is ... **opposite** the right angle. The **adjacent** side (a) ... is next to ... **angle** (θ); the **opposite** side (b) is across from ... angle (θ).”



Inverse Trig Functions are:

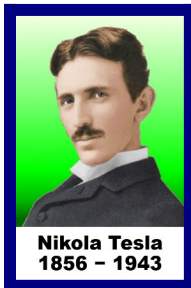
$$\theta = \text{Cos}^{-1}(a/r)$$

$$(0^\circ \leq \theta \leq 180^\circ)$$

$$\theta = \text{Sin}^{-1}(b/r)$$

$$(-90^\circ \leq \theta \leq 90^\circ)$$

Turns Ratio Formula EM Color



Nikola Tesla
1856 - 1943

In 1834, Emil Lenz discovered Lenz's Law for a wire loop as:

$$\mathcal{E} = -\Delta\Phi_B/\Delta t$$

An **electromotive force** (\mathcal{E} - emf) & **current** flow oppose a change in magnetic flux ($\Delta\Phi_B$) supporting Maxwell's 3rd Law (1862).

$$\mathcal{E} = \oint (\mathbf{E} \cdot d\boldsymbol{\ell}) \quad \& \quad \Phi_B \equiv \iint (\mathbf{B} \cdot \hat{\mathbf{n}}_s) ds$$

In 1882, L Gaulard & J Gibbs first used transformers to reduce Alternating Current (AC) losses. N Tesla

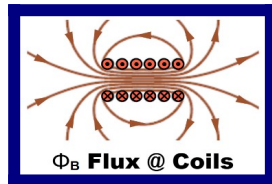
& G Westinghouse subsequently made improvements in AC motors & other support equipment.

In AC transformers, voltage change (V_i) around the loop equals the emf ($V_i = \mathcal{E}$). With coil count (n_i), Lenz's Law becomes the Turns Ratio Formula:

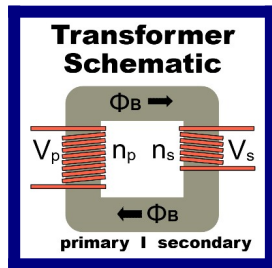
$$-n_i(d\Phi_B/dt) = V_i \quad \Leftrightarrow \quad -d\Phi_B/dt = V_i/n_i \quad \Leftrightarrow \quad n_p/V_p = n_s/V_s$$

In 1896, Westinghouse applied this formula to electrically power Buffalo, New York from Niagara Falls 26 miles (42 km) away.

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Φ_B Flux @ Coils



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