Why Can't We Exceed Light Speed?

Introduction

About Special Relativity (SR), the public often wonders why Albert Einstein (1879 – 1955) (ref [1]) put a speed limit on the Universe (ref [2]). This speed limit is the speed of light in a vacuum & is commonly assigned the variable constant (*c*).

When asked: "Why can't a spaceship exceed the speed of light?"
The answer is: "Rocket mass increases to infinity as light speed is approached."

This article let's the physicist give an alternate answer to the one above. BTW, the speed limit of the Universe is about (refs [3] & [4]):

675,000,000 miles / hour
1079,000,000 kilometers / hour

It'll be awhile before your city's finest start giving speeding tickets for these speeds! The value of (*c*) is a fundamental constant of nature. One may wonder why this constant has the value it does? From Physics, currently: "We don't know." (ref [5]).

Rocket Mass (Newton vs. Einstein)

In SR, the dynamic rocket mass with rest mass (m_0) moving with velocity ($\boldsymbol{u}$) is taken from its SR momentum ($\boldsymbol{p}_{\text{Einstein}}$) definition (refs [6] & [7]) in view of its Newtonian momentum ($\boldsymbol{p}_{Newton}$):

$$\boldsymbol{p}_{Newton} \equiv m_0 \boldsymbol{u} \qquad \text{verses} \qquad \boldsymbol{p}_{Einstein} \equiv \frac{m_0 \boldsymbol{u}}{\sqrt{1 - \boldsymbol{u}^2/c^2}} \qquad \text{then}$$

$$(m_0)_{Newton} \rightarrow \left(\frac{m_0}{\sqrt{1 - \boldsymbol{u}^2/c^2}} \right)_{Einstein}$$

The spaceship rest mass (m_0), is measured when the spaceship is stationary ($\boldsymbol{u} = 0$). As the spaceship velocity ($\boldsymbol{u}$) approaches the speed of light (*c*), its dynamic mass does increase without limits. However, no more molecules or atoms are popping into existence to increase rocket rest mass (m_0) which the above verbal answer implies.

Rocket Acceleration Transformation

As a rocket approaches the speed of light (*c*), it's acceleration can be transformed from the view point of the rocket to that of an observer back @ it's launch base. Regardless of the spaceship's "conventional" rocket engines, it will not be able to "break" the barrier of light speed. The acceleration vector transformation equation can be derived in vector form (refs [6] & [9]). Then, the acceleration of a spaceship near light speed can be expressed relative to the base observer. This equation will show which dominant terms prevent a spaceship from overcoming (*c*). Here are the space-time Lorentz transformations in vector form (refs [6] & [7] & [8]):

$$t' = \gamma_v \left[t - \frac{(\boldsymbol{r} \cdot \boldsymbol{v})}{c^2} \right] \qquad \boldsymbol{r}' = \boldsymbol{r} + \gamma_v \boldsymbol{v} \left[\frac{(\boldsymbol{r} \cdot \boldsymbol{v})}{c^2} \sigma_v - t \right]$$

with auxiliary variables defined (ref [9]):

$$\gamma_v \equiv \frac{1}{\sqrt{1 - \boldsymbol{v}^2/c^2}} \qquad \sigma_v \equiv (1 - 1/\gamma_v)(c^2/\boldsymbol{v}^2) = \frac{1}{1 + 1/\gamma_v}$$

Hendrik Lorentz derived his namesake equations in 1899 (ref [10]) to maintain that light speed in a vacuum is measured @ the value (*c*) regardless of reference frame speed (***v***). To express the transformation of derivatives @ velocity (***v***), take the derivative of the Lorentz transformation equations with respect to (wrt) time (*t*) before the transformation. Hold the transformation velocity (***v***) constant wrt time (*t*) (refs [7] & [8]).

$$\frac{dt'}{dt} = \gamma_v \left[1 - \frac{(\boldsymbol{u} \cdot \boldsymbol{v})}{c^2} \right] \qquad \boldsymbol{u}' \equiv \frac{d\boldsymbol{r}'}{dt'} = \frac{dt}{dt'} \frac{d\boldsymbol{r}'}{dt} = \frac{1}{\delta_{uv}} \left[\boldsymbol{u} + \gamma_v \boldsymbol{v} \left(\frac{(\boldsymbol{u} \cdot \boldsymbol{v})}{c^2} \sigma_v - 1 \right) \right]$$

with auxiliary variable (ref [11]):

$$\delta_{uv} \equiv \frac{dt'}{dt} = \gamma_v \left[1 - \frac{(\boldsymbol{u} \cdot \boldsymbol{v})}{c^2} \right]$$

For the general acceleration transformation (***a'***), the derivative of the velocity transformation (***u'***), is taken again holding the transformation velocity (***v***) constant wrt time (*t*).

$$\boldsymbol{a}' \equiv \frac{d\boldsymbol{u}'}{dt'} = \frac{dt}{dt'} \frac{d\boldsymbol{u}'}{dt} = \frac{1}{\delta_{uv}^2} \left[\boldsymbol{a} - \frac{\gamma_v (\boldsymbol{a} \cdot \boldsymbol{v})}{\delta_{uv} c^2} (\boldsymbol{v} \sigma_v - \boldsymbol{u}) \right]$$

The above equation transforms the acceleration vector (***a***) of a particle moving with velocity vector (***u***) to a reference frame moving @ velocity (***v***). The preceding Lorentz equations show the interplay between the length contraction term (σ_v) & time dilation term (δ_{uv}) (ref [11]). The objective of the extra terms is to maintain the requirement of nature that light speed (*c*) be measured @ a constant value across reference frame transformations.

Observed Acceleration Near (*c*)

For our spaceship, the acceleration vector ($\boldsymbol{a}_t$) & time derivative of time (δ_{uv}) are transformed from the spaceship's rest frame where ($t = \tau$), ($\boldsymbol{u} = 0$), ($\delta_{0v} = \gamma_v$), ($\boldsymbol{a}_t = \boldsymbol{a}_\tau$). If the spaceship is moving with velocity (***v***) wrt the base observer, then the transformation velocity is (-***v***) from the ship to base. Across the Lorentz transformations, the base observer will measure ***all*** physics occurring in the fast moving spaceship such that the speed of light (c) is constant (ref [11]).

$$\boldsymbol{a}'\big|_{\boldsymbol{u}=0,t=\tau} = \frac{1}{\gamma_v^2}\left[\boldsymbol{a}_\tau - \boldsymbol{v}\frac{(\boldsymbol{a}_\tau \cdot \boldsymbol{v})\sigma_v}{c^2}\right]$$

$$\delta_{0v} = \frac{dt'}{dt}\bigg|_{\boldsymbol{u}=0,t=\tau} \equiv \frac{dt'}{d\tau} = \gamma_v \qquad \left(\frac{d\tau}{dt'}\right)^2 = \frac{1}{\gamma_v^2}$$

For our case,

$$\boldsymbol{a}'(t')\big|_{\boldsymbol{u}=0,t=\tau} = \frac{d}{dt'}\frac{d}{dt'}\boldsymbol{r}'(t')\bigg|_{\boldsymbol{u}=0,t=\tau} = \left(\frac{d\tau}{dt'}\right)^2\left[\boldsymbol{a}_\tau - \boldsymbol{v}\frac{(\boldsymbol{a}_\tau\cdot\boldsymbol{v})\sigma_v}{c^2}\right] = \left(\frac{d\tau}{dt'}\right)^2\boldsymbol{a}_\tau - \boldsymbol{v}\frac{(\boldsymbol{a}_\tau\cdot\boldsymbol{v})\sigma_v}{\gamma^2 c^2}$$

Then,

$$\boldsymbol{a}'(t')\big|_{\boldsymbol{u}=0,t=\tau} = \left(\frac{d\tau}{dt'}\right)^2\boldsymbol{a}_\tau + \frac{1}{c^2}\boldsymbol{\Theta}(\boldsymbol{a}_\tau,\boldsymbol{v})$$

The function ($\boldsymbol{\Theta}(\boldsymbol{a}_\tau,\boldsymbol{v})/c^2$) simply collects lower magnitude terms for the equality.

An Interpretation

For the rocket acceleration from perspective of the observer's primed-reference frame at launch (ref [11]), then

$$\boldsymbol{a}'_{observer} = \boldsymbol{a}'(t')\big|_{\boldsymbol{u}=0,t=\tau} = \left(\frac{d\tau}{dt'}\right)^2\boldsymbol{a}_\tau + \frac{1}{c^2}\boldsymbol{\Theta}(\boldsymbol{a}_\tau,\boldsymbol{v})$$

Substituting:

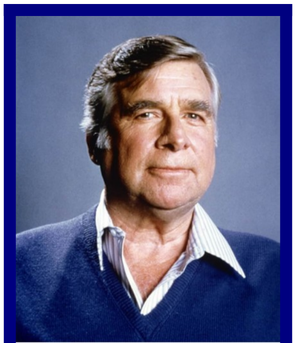
$$\boldsymbol{a}'_{observer} = \left(1-\frac{\boldsymbol{v}^2}{c^2}\right)\boldsymbol{a}_\tau + \frac{1}{c^2}\boldsymbol{\Theta}(\boldsymbol{a}_\tau,\boldsymbol{v})$$

That's the double derivative of time transformation from the spaceship's rest frame, time dilation squared because acceleration is per second per second. The accelerating rocket engine's ability to eject photons or to perform the chemical reactions to eject ions / propellant, slows as the electromagnetic communication between atoms in the rocket engine slows. All aspects of the rocket engine function more slowly to accommodate the earth observer's perception / measurements of the spaceship's actions slowing near (c). From the base observer, the closer a spaceship's velocity is to (c), the more the rocket's engine performance degrades & the spaceship simply coasts @ near the speed of light (refs [11] & [12]).

Breaking the Light Speed Barrier

To illustrate the maximum speed limit (c) of space, this article derived the rocket classical SR acceleration equations that evaluate the velocity (***u***(*t'*)) & acceleration (***a***(*t'*)) of a rocket approaching the speed of light. The equations give values from the rocket's static launch reference frame in time (*t'*). These equations are also fundamental in modeling futuristic space travel with Numerical Analysis as used in very early Microsoft Flight Simulator (1980's, refs [13] & [14]). The necessary algebra has existed (refs [6] & [7] & [9]) for decades.

The most difficult math of classical SR is a ***square root***! Once a rocket with mass is expressed traveling @ less than light speed, it will never break the SR speed limit of (*c*) no matter the finite magnitude of rocket acceleration & rocket engine performance.



G Roddenberry 1921 – 1991

As Gene Roddenberry (ref [15]) had envisioned, the USS Enterprise (NCC−1701) (ref [16]) of Star Trek lore incorporated anti-matter in its "warp speed" engines using General Relativity (GR). This theory was required to exceed the speed of light limit, a demarcation that is rigorously enforced in the acceleration transformation equation of SR derived above. Science Fiction (SciFi) is not concerned with this SR limit, since spaceships will be warping space per GR to travel @ speeds toward & above (*c*).

References

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