Quick Study: Four Metric Conversions

Introduction

This article gives conversions for 4 customary to metric quantities that I perform mentally almost daily. Two conversion quantities are for length, one is for weight & one is for volume. Each ***approximate*** conversion is given with the ***exact*** conversion & its derivation using the *factor-label* method (ref [1]), also known as the *chain-link* method (ref [2]). For example, "in 1988, Congress enacted legislation (Public Law 00-418) to establish" that one inch equal 2.54 centimeters ***exactly*** (ref [3]). That's great!!! Then, many other metric conversions of lengths, be it feet, yards, or miles are derived from that value ***exactly***. In addition, a 1991 "executive order directed all Federal agencies to implement 'metrification' to the extent economically feasible by September 30, 1992." We are still waiting for that!!!

Metric Prefixes

The Metric System (International System of Units – SI) is principally governed by Système International d'Unités in Sèvres, France. The metric system was decreed by French law in 1799, during the French Revolution to mark a new beginning (ref [4]). In 1793, the French even tried a metric clock (ref [5]), based on "a 10-hour day, with 100 minutes per hour, and 100 seconds per minute." High noon was at 5 o'clock! "The system proved unpopular … replacing every clock and watch in the country was" too costly.

The French decided on these metric prefixes to the right which should last into the distant future (ref [6]). We now have a US national budget totaling trillions of dollars ($1T = $10^{+12}). For the fiscal conservatives, we toss around tera-dollars ($1T) like "terror-money". Soon, our national spending will be in PetaBucks ($1P = $10^{+15}). That sounds like a fund-raising mnemonic for a charity!

SI Metric Prefixes

name	abr	factor		ordinal
quetta	Q	1.0E+30	10^{+30}	Nonillion
ronna	R	1.0E+27	10^{+27}	Octillion
yotta	Y	1.0E+24	10^{+24}	Septillion
zetta	Z	1.0E+21	10^{+21}	Sextillion
exa	E	1.0E+18	10^{+18}	Quintillion
peta	P	1.0E+15	10^{+15}	Quadrillion
tera	T	1.0E+12	10^{+12}	Trillion
giga	G	1.0E+09	10^{+09}	Billion
mega	M	1.0E+06	10^{+06}	Million
kilo	K	1000	10^{+03}	Thousand
hecto	h	100	10^{+02}	Hundred
deka	da	10	10^{+01}	Ten
–	–	1	10^{+00}	One
deci	d	0.1	10^{-01}	Tenth
centi	c	0.01	10^{-02}	Hundredth
milli	m	0.001	10^{-03}	Thousandth
micro	μ	1.0E-06	10^{-06}	Millionth
nano	n	1.0E-09	10^{-09}	Billionth
pico	p	1.0E-12	10^{-12}	Trillionth
femto	f	1.0E-15	10^{-15}	Quadrillionth
atto	a	1.0E-18	10^{-18}	Quintillionth
zepto	z	1.0E-21	10^{-21}	Sextillionth
yocto	y	1.0E-24	10^{-24}	Septillionth
ronto	r	1.0E-27	10^{-27}	Octillionth
quecto	q	1.0E-30	10^{-30}	Nonillionth

Mass & Force

Unless you're a scientist or engineer, you should not let the distinction between mass & force discourage you from learning the metric system. To do the "book keeping", the units of mass include pound-mass, slugs & kilograms; the units of force are pound-force

& Newtons (ref [7]). When someone mentions mass & weight in the same sentence, refer them to the nearest College Physics or Engineering Department. For all other instances, a kilogram is about double a pound (see below).

Calculating Conversion Factors

Conversion factors are cumbersome but necessary quantities which, @ times, must be accurate or much effort can be destroyed (ref [8]). In 1999, a NASA satellite which should have entered a Mars orbit, instead crashed on the planet's surface. The mishap was due to a metric conversion error made before launch.

Below is given the standard way (ref [10] & ref [11]) conversion factors are calculated (usually on a spreadsheet). Notice the multiplication & division is made such that intermediate units of measure all cancel out. When coding, the programmer can leave the initial factors in the code. The compiler will perform as much arithmetic as possible, storing the floating-point results as a constant before code execution. These fundamental values in the code can then be easily checked or updated.

Units of Measure are 100% of the Answer

"Any number is an approximation of any other number!" Regardless of how much effort is put into a calculation, if the units of measure supplied are incorrect, then the efforts are useless! Rules will be briefly reviewed here in how to carry units through most math calculations (ref [1] & [9]).

1) **Math functions:** for most math functions, including: *sin*(*θ*), *Arcsin*(*x*), *log*(*x*), *exp*(*x*), the input parameter(s) must be ratios without units. BTW, trigonometric angles are expressed in radians, a ratio of arc length divided by arc radius.
2) **Exponential powers:** for values with units that are squared, cubed, evaluated in a square root, the exponent is applied to the value & passed to the units.
3) **Multiplication, division:** when two numbers are multiplied, the units are multiplied also. An often occurrence, equivalent units which are in the numerator & denominator cancel, eliminate each other & should not be carried forward.
4) **Addition, subtraction:** when values are added or subtracted, their units ***must*** agree. If not, multiplying the correct conversion factors are necessary before the addition / subtraction.

Examples:

An area of one square inch is about 6.5 square centimeters.

$$\left(\frac{2.54\,cm}{1\ \text{in}}\right)^2 = \left(2.54^2\frac{cm^2}{\text{in}^2}\right) = 6.4516\frac{cm^2}{\text{in}^2}$$

A temperature of 25° Celsius is 77° Fahrenheit exactly.

$$\left[25\,^\circ C \times \left(\frac{180\,^\circ F}{100\,^\circ C}\right)\right] + 32\,^\circ F \;=\; 77\,^\circ F$$

% Error Calculations

By request, percent errors are supplied for the mental estimations of the customary ↔ metric conversion evaluations. The definition for % error between an "exact" value and an "estimated" value is (ref [12]):

$$E_{\%} \;=\; \frac{\left|x_{exact} - x_{estimate}\right|}{x_{exact}} 100\,\%$$

In % error determinations, the most accurate value is usually in the denominator, and the ***exact*** conversion value is used here. Negative percent errors should be defined when the error value is given. In these calculations, an absolute value of an evaluation difference is used; all negative values are eliminated. A single % error value applies to a conversion both ways, i.e., a reduction or increase in value. Values of % error differ slightly with direction of conversion. The higher % error value is always listed here.

Four Common Conversion Factors

The exact value for each of the 4 customary ↔ metric quantities is given (ref [1]). An approximation calculation is also described that lets one mentally estimate the metric conversion. An upper limit % error is also stated for each unit conversion.

A yard is about equivalent to a meter. (10% error)

$$\left(\frac{1\,\text{in}}{2.54\,cm}\right)\left(\frac{1\,ft}{12\,\text{in}}\right)\left(\frac{1\,yd}{3\,ft}\right)\left(\frac{100\,cm}{1\,m}\right) \;=\; \left(\frac{100\,yd}{91.44\,m}\right) \;\approx\; 1.09\frac{yards}{meter} \;\approx\; 1\frac{yard}{meter}$$

A kilometer is about half a mile. (25% error)

A mile is about twice a kilometer.

$$\left(\frac{1\,\text{in}}{2.54\,cm}\right)\left(\frac{1\,ft}{12\,\text{in}}\right)\left(\frac{1\,mile}{5280\,ft}\right)\left(\frac{100\,cm}{1\,m}\right)\left(\frac{1000\,m}{1\,km}\right) \;=\; \left(\frac{100000\,mile}{160934\,km}\right) \;\approx\; \frac{1}{2}\frac{miles}{kilometer}$$

A kilogram is about twice a pound. (11% error)

A pound is about half a kilogram.

$$\left(\frac{1\,avoirdupois\,lb_{mass}}{0.45359237\,kg}\right)\left(\frac{1\,lb_{force}}{1\,avoirdupois\,lb_{mass}}\right) \;=\; \left(\frac{1\,lb_{force}}{0.45359237\,kg}\right) \;\approx\; 2\frac{pounds}{kilogram}$$

A liter is about equivalent to a quart. (6% error)

$$\left(\frac{1\,qt}{57.75\,\text{in}^3}\right)\left(\frac{1\,\text{in}}{2.54\,cm}\right)^3\left(\frac{1\,cc}{1\,mL}\right)\left(\frac{1000\,mL}{1\,L}\right) \;=\; \left(\frac{1000\,qt}{946.353\,L}\right) \;\approx\; 1.06\frac{quart}{liter} \;\approx\; 1\frac{quart}{liter}$$

Another Mile ↔ Kilometer Conversion Method

From the conversion estimations above, miles ↔ kilometers calculation has the most inaccuracy with about a 25% error. A more exact estimation exists using the “five-eighths” rule (ref [13]). To convert kilometers to miles, “multiply by 5 & divide by 8”. For miles to kilometers “multiply by 8 & divide by 5”. Instead of ½ = 0.5, the “five-eighths” factor here is decimal ⅝ = 0.625 ≈ 0.62137 with an error of about 0.58%, a less than a one-percent error, a much better estimate.

For me, I’ll forego the mental gymnastics & do my math by a factor of 2 while driving ***without*** a wreck. BTW, if you forget the numbers 5 & 8, notice how 50 mph equals 80 kph on your US car’s speedometer, while driving of course!

Conclusion

Every generation, or so, the US should try to convert to the metric system. It’s something people do for their posterity (ref [14]). It makes society more efficient, allows for more artists & pure scientists, and less “burger flippers” for the same amount of social effort. Just think, all of the values & mathematical calculations discussed above would be in a “How Bad Was It?” museum. I won’t live to see the event, but I’m sure one day the US will make the conversion.

References

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