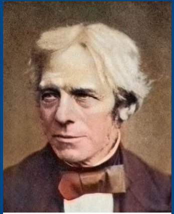
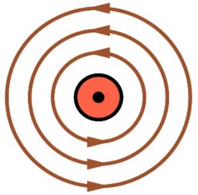
DC Electric Motors & Wire Coils

EM Color



M Faraday
1791 – 1867

By 1820, scientists noticed that a compass needle aligned circumferentially around a direct current (DC) in a conductor. Current flow direction sets the compass "North" orientation.

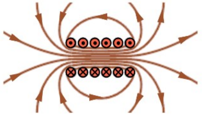


B-Field @ Wire

Michael Faraday envisioned magnetic (B-field) lines surrounding a DC wire. In Maxwell's Equations, a magnetostatic 4th Law gives a circular B-field. If a wire with DC current (I) is wrapped around an iron core (n_c) times, a bar magnet exists.

4th Law: $\oint(\mathbf{B} \cdot d\ell) = n_c\, \mu_0 \iint(\mathbf{J} \cdot \hat{\mathbf{n}}_s)\, ds$ & $I = \iint(\mathbf{J} \cdot \hat{\mathbf{n}}_s)\, ds$

$\oint(\mathbf{B}_r \cdot d\ell) = |\mathbf{B}_r|\,(2\pi r) = n_c\, \mu_0\, I$ ⇨ $|\mathbf{B}_r| = (n_c\, \mu_0\, I)\,/\,(2\pi r)$



B-Field @ Coils

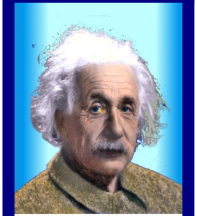
The 3rd Law gives impeding Voltage (V) from change in applied magnetic flux ($\Delta\Phi_B$) through magnet coils. The flux increases due to bar magnet alignment.

3rd Law: $\oint(\mathbf{E} \cdot d\ell) = -d/dt \iint(\mathbf{B} \cdot \hat{\mathbf{n}}_s)\, ds$

$V = n_c \oint(\mathbf{E} \cdot d\ell)$ & $\Phi_B \equiv \iint(\mathbf{B} \cdot \hat{\mathbf{n}}_s)\, ds$ ⇨ $V = -n_c\, \Delta\Phi_B/\Delta t$

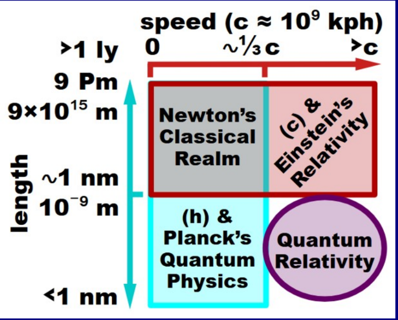
Wikipedia.org

Einstein Extremes



Albert Einstein
1879 – 1955

Since 1905, Einstein has made revolutionary advances in Physics. He refined Quantum ideas & invented Relativity.



From his photon theory, light is transmitted in "quanta" of Energy (E_0):

$$E_0(\nu) = h\nu$$

From Newton in 1687, 3-vector lengths ($r^2 \equiv \mathbf{r}\bullet\mathbf{r}$) are constant in spatial rotations. Mass (Δm_0) & Time (Δt) are scalars.

3D Momentum: $\mathbf{p} \equiv m_0\mathbf{u}$ **Kinetic Energy:** $E = \frac{1}{2}m_0u^2$

With Special Relativity, we live in 4D spacetime, clock rates vary. 4-vector length ($R^\mu R_\mu \equiv (ct)^2 - r^2$) is conserved in spatial rotation, velocity boost.

Lorentz Factor: $\gamma \equiv 1/\sqrt{1-u^2/c^2}$ with 3D Velocity ($\mathbf{u}$)

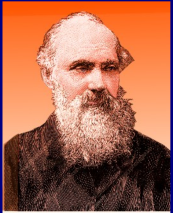
3D Momentum: $\mathbf{p} \equiv \gamma m_0\mathbf{u}$ **Total Energy:** $E \equiv \gamma m_0c^2 \approx m_0c^2 + \frac{1}{2}m_0u^2 + \cdots$

4-Position: $R^\mu \equiv (ct, \mathbf{r})$ **4-Momentum:** $P^\mu \equiv (E/c, \mathbf{p})$

Einstein helped define extremes setting challenges in Modern Physics.

Wikipedia.org

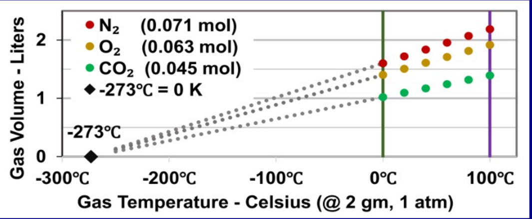
Finding Absolute Zero



W Thomson
1824 – 1907

From the thermometer invented in 1714, ideal gas behavior was given in Charles's Law by 1787:

$$T_1/V_1 = T_2/V_2$$



The Ideal Gas Law approximates certain gases @ molecular mass (m), temperature (T_i), volume (V_i) w/restrictions:

a) average Kinetic Energy $\equiv \langle \frac{1}{2}mv^2 \rangle \propto T$.

b) point particles w/no interactions.

c) constant motion w/elastic collisions.

In 1848, William Thomson (Lord Kelvin) proposed an Absolute Zero (0 K) @ -273°C extrapolated from near ideal gas data between 0°C & 100°C.

local		add ⇨ ⇦ subtract	absolute	
Celsius	(°C)	±273.15	Kelvin	(K)
Fahrenheit	(°F)	±459.67	Rankine	(R)

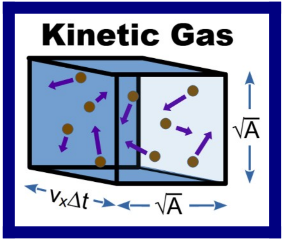
At Absolute Zero, particle thermal kinetic energy stops. A "Zero-Point Energy" of random quantum motion remains:

$$\Delta x_i \Delta p_i = \frac{1}{2}\hbar \quad \& \quad \Delta E \Delta t = \frac{1}{2}\hbar$$

Wikipedia.org

Ideal Gas Law

D Bernoulli
1700 – 1782

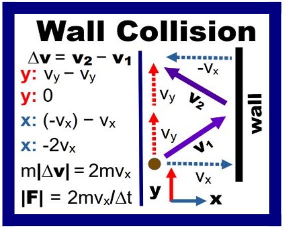


In 1738, Daniel Bernoulli, published his kinetic theory of gases. Molecules exert pressure through elastic collisions with container walls. For a gas @ absolute temperature (T) w/particles having molecular mass (m), speed (v), kinetic energy ($KE \equiv \frac{1}{2}mv^2$), the theory requires:

a) **Boltzmann constant (k_B) relates** $\langle KE\rangle \propto \langle mv^2\rangle = k_BT$.
b) **constant motion w/elastic collisions.**
c) **point particles w/o interactions.**

Pressure (p_A) on container wall w/area (A) is force (F) per area of count (N) particles w/reversed velocity ($2mv_x$). An average is taken over time (Δt).

$\frac{1}{2}N$ **particles collide w/wall** A **in time** Δt ($v_x > 0$)
Container has volume (V): $V = Av_x\Delta t$ ⇨ $1/A = v_x\Delta t/V$
$p_A = \frac{1}{2}N\langle|\mathbf{F}|/A\rangle = \frac{1}{2}N\langle(2m\mathbf{v}/\Delta t)\cdot(\mathbf{v}\Delta t/V)\rangle = N\langle mv^2\rangle/V$



Ideal Gas Law: $N\langle mv^2\rangle = N k_BT$ ⇨ $p_AV = N k_BT$

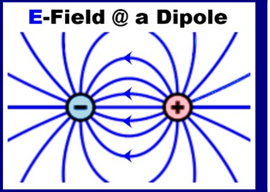
Wikipedia.org

Lorentz Force Equation

EM Color

HA Lorentz
1853 – 1928

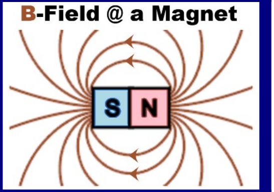
In 1895, Hendrik Lorentz added the E-field to Oliver Heaviside's magnetic force to state his final electromagnetic (EM) force equation:



$$\mathbf{F}_t = q_t\,[\mathbf{E}(\mathbf{x}_t) + \mathbf{v}_t\times\mathbf{B}(\mathbf{x}_t)]$$

$\mathbf{F}_t$ – 3D vector force (N – Newtons)
$\mathbf{x}_t$ – 3D vector location (m – meters)
$\mathbf{v}_t$ – 3D vector velocity (m/s – meter/second)
q_t – scalar charge (C – Coulomb) (1 C ≈ – 6.2×10^{18} e^-)

$\mathbf{E}(\mathbf{x}_t)$ – 3D electric field vector (N/C – Volts/meter). E-fields define EM forces assuming near infinite propagation speeds & static states. E-fields undergo work, store & expend energy.



$\mathbf{B}(\mathbf{x}_t)$ – 3D magnetic field vector (T – Tesla). B-fields set corrections due to a finite EM propagation speed ($c \approx 10^9$ ft/s ≈ 10^9 kph). B-fields do NO work (W_B).

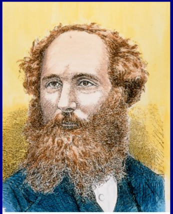
$$\mathbf{F}_B \equiv q_t(\mathbf{v}_t\times\mathbf{B}) \;\Rightarrow\; \mathbf{F}_B \perp \mathbf{v}_t \;\;\&\;\; \mathbf{F}_B \perp \mathbf{B}$$

$$d\boldsymbol{\ell}_t \approx \mathbf{v}_t\,\Delta t \;\Rightarrow\; W_B \equiv (\mathbf{F}_B\cdot d\boldsymbol{\ell}_t) = (\mathbf{F}_B\cdot\mathbf{v}_t)\Delta t = q_t[(\mathbf{v}_t\times\mathbf{B})\cdot\mathbf{v}_t]\Delta t = 0 \;\Rightarrow\; W_B = 0$$

Wikipedia.org

Maxwell's Equations

EM Color

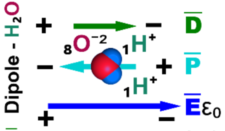


JC Maxwell
1831 – 1879

By 1862, James Maxwell published his equations that codified known data of classical Electromagnetism. By 1891, work of Oliver Heaviside was credited with their final form.

1st) $\nabla \bullet \mathbf{D} = \rho$	$\oiint (\mathbf{D} \cdot \hat{\mathbf{n}}_s) ds = \iiint \rho dv$
2nd) $\nabla \bullet \mathbf{B} = 0$	$\oiint (\mathbf{B} \cdot \hat{\mathbf{n}}_s) ds = 0$
3rd) $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$	$\oint (\mathbf{E} \cdot d\ell) = -d/dt\, [\iint (\mathbf{B} \cdot \hat{\mathbf{n}}_s) ds]$
4th) $\nabla \times \mathbf{H} = \mathbf{J} + \partial \mathbf{D}/\partial t$	$\oint (\mathbf{H} \cdot d\ell) = \iint (\mathbf{J} \cdot \hat{\mathbf{n}}_s) ds + d/dt\, [\iint (\mathbf{D} \cdot \hat{\mathbf{n}}_s) ds]$

Dielectrics



$\overline{\mathbf{D}}$ - Metric Displacement (C/m^2)
$\overline{\mathbf{P}}$ - H_2O Dipole Density (C/m^2)
$\overline{\mathbf{E}}$ - Applied Electric Field (V/m)

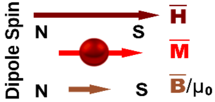
These equations were developed in terrestrial settings far from the pure vacuum of outer space. In a vacuum ⇨ $\kappa = 0$ ⇨ $\mathbf{D} = \varepsilon_0 \mathbf{E}$ **&** $\mathbf{H} = \mathbf{B}/\mu_0$. **For a gas** $|\kappa| < 1$:

$$\mathbf{D} \equiv \varepsilon \mathbf{E} \quad \& \quad \varepsilon / \varepsilon_0 \equiv (1-\kappa) \Rightarrow \mathbf{P} = -\kappa \varepsilon_0 \mathbf{E} \quad \& \quad \varepsilon_0 \mathbf{E} = \mathbf{D} - \mathbf{P}$$

$$\mathbf{H} \equiv \mathbf{B}/\mu \quad \& \quad \mu_0 / \mu \equiv (1+\kappa) \Rightarrow \mathbf{M} = \kappa/\mu_0 \mathbf{B} \quad \& \quad \mathbf{B}/\mu_0 = \mathbf{H} - \mathbf{M}$$

To measure Electric (E) & Magnetic (B) fields in 1800's labs, 1st order linear Dielectric (P) & Magnetic (M) dipoles were included to match lab results.

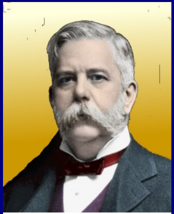
Magnetics



$\overline{\mathbf{H}}$ - Metric Magnetic Field (A/m)
$\overline{\mathbf{M}}$ - Magnetization (A/m)
$\overline{\mathbf{B}}$ - Applied Magnetic Flux (T)

Wikipedia.org

Moving Electricity Efficiently



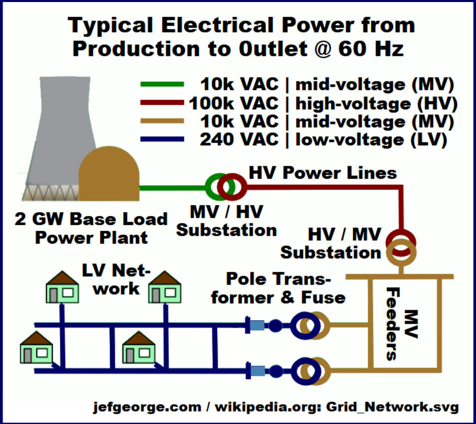
Westinghouse
1846 – 1914

In 1896, [Westinghouse Electric] gave power to Buffalo, New York 26 miles (42 km) from Niagara Falls. Based on [Ohm's Law], the design cut heat loss (P_{Loss}↓) by reducing the transmission current (I_{Tran}↓) across [power line] resistance (R_{Tran}●).

$\eta_{eff} \equiv (P_{Load}\bullet)/[(P_{Load}\bullet)+(P_{Loss}\downarrow)]$

$(P_{Loss}\downarrow) = (R_{Tran}\bullet)(I_{Tran}\downarrow)^2$

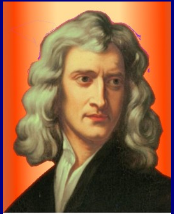
$(P_{Load}\bullet) = (V_{Tran}\uparrow)(I_{Tran}\downarrow)\ \eta_{eff}$



For a set power (P_{Load}●), the reduced current (I_{Tran}↓) cut losses (P_{Loss}↓), driving efficiency (η_{eff}) toward unity. The transmission voltage (V_{Tran}↑) was [stepped up] by [transformers] to meet the required load (P_{Load}●).

[Wikipedia].org

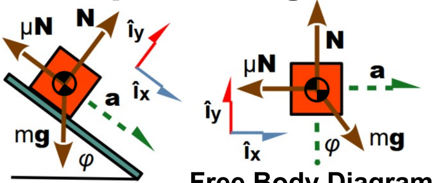
Newton's Laws of Motion



Isaac Newton
1642 – 1726

In 1687, Isaac Newton published *Principia* in Latin, because most great tomes were from Roman Antiquity over 1000 years before. His book described gravity & 3 Laws of Motion:

Example: Sliding Block



Free Body Diagram

N - normal force
a – acceleration
g – gravity
m - block mass
μ – friction factor
φ – incline angle

y: $ma_y = N - mg\cos\varphi = 0$
y: $N = mg\cos\varphi$
x: $ma_x = mg\sin\varphi - \mu N$
x: $a_x = g(\sin\varphi - \mu\cos\varphi)$
x: $a_x = g\cos\varphi(\tan\varphi - \mu)$

$\sum F_i = ma$ Block Slides @ $\varphi > \mathrm{Atan}(\mu)$

1) "Every object moves in a straight line unless acted upon by a force."

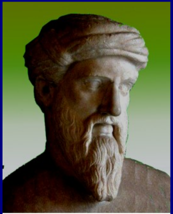
2) For a body with scalars mass (m) & acceleration (a), force (F) is: F = ma. The scalar magnitude of 3D vector (a) is: $a = |\mathbf{a}| \equiv (\mathbf{a}\bullet\mathbf{a})^{1/2}$ ⇨ **F** = m**a**

3) "For every action, there is an equal & opposite reaction."

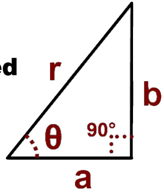
Newton's Laws adapted for Center-of-Mass Analyses: $\sum \mathbf{F}_i = m\mathbf{a}$

Wikipedia.org

Pythagorean Theorem



Pythagoras
~570 – ~495 BCE



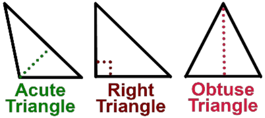
Around 530 BCE, Pythagoras of Samos founded his ascetic school in Kroton, Italy. Its ideas influenced Aristotle, Plato & Renaissance thinkers. By 850 CE, Trigonometry (Trig) Functions used the theorem named for him.

$a^2 + b^2 = r^2$ ⇨ $\cos^2\theta + \sin^2\theta = 1$

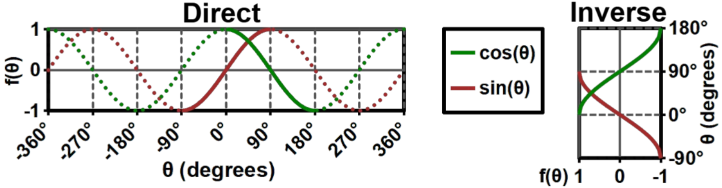
with:

$\cos\theta \equiv a/r$ $\sin\theta \equiv b/r$

$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$ $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$



The theorem only applies to right triangles. "The hypotenuse (r) … is … opposite the right angle. The adjacent side (a) … is next to … angle (θ); the opposite side (b) is across from … angle (θ)."



Inverse Trig Functions are:

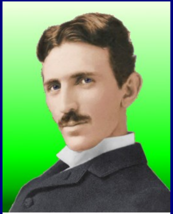
$\theta = \text{Cos}^{-1}(a/r)$
$(0° \le \theta \le 180°)$

$\theta = \text{Sin}^{-1}(b/r)$
$(-90° \le \theta \le 90°)$

Wikipedia.org

Turns Ratio Formula

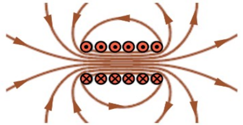
EM Color



Nikola Tesla
1856 – 1943

In 1834, Emil Lenz discovered Lenz's Law for a wire loop as:

$$\mathcal{E} = -\Delta\Phi_B / \Delta t$$



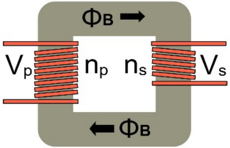
Φ_B Flux @ Coils

An electromotive force ($\mathcal{E}$ – emf) & current flow oppose a change in magnetic flux ($\Delta\Phi_B$) supporting Maxwell's 3rd Law (1862).

$$\mathcal{E} = \oint(\mathbf{E} \cdot d\ell) \quad \& \quad \Phi_B \equiv \iint(\mathbf{B} \cdot \hat{\mathbf{n}}_s)\, ds$$

In 1882, L Gaulard & J Gibbs first used transformers to reduce Alternating Current (AC) losses. N Tesla & G Westinghouse subsequently made improvements in AC motors & other support equipment.

Transformer Schematic



In AC transformers, voltage change (V_i) around the loop equals the emf ($V_i = \mathcal{E}$). With coil count (n_i), Lenz's Law becomes the Turns Ratio Formula:

$$-n_i(d\Phi_B/dt) = V_i \;\Rightarrow\; -d\Phi_B/dt = V_i/n_i \;\Rightarrow\; n_p/V_p = n_s/V_s$$

In 1896, Westinghouse applied this formula to electrically power Buffalo, New York from Niagara Falls 26 miles (42 km) away.

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