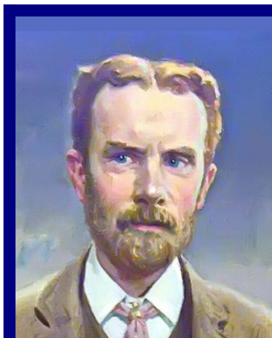
Maxwell's Equations Demystified

Introduction

From 1785 with Charles-Augustin de Coulomb (1736 – 1806) (ref [1]) in establishing electrostatics as an inverse square law, to André-Marie Ampère (1775 – 1836) (ref [2]) in relating electric current & magnetic force in 1820, to the creation of voltage from change in magnetic flux of Michael Faraday (1791 – 1867) (ref [3]), to Johann Carl Friedrich Gauss (1777 – 1855) (ref [4]) describing magnetostatics around a current carrying wire in 1835, the discoveries describing the phenomena of electromagnetism (EM) captivated the academics during the first half of the 19th century.

BTW, Monsieur Coulomb was a Frenchman, so his name is pronounced "COO-lam" (ref [5]). Monsieur Ampère was also a Frenchman, so his name is pronounced "am-PEER" (ref [6]). Herr Gauss was German & his name rhymes with "house" (ref [7]).



O Heaviside
1850 – 1925

Like the mysteries of Quantum Mechanics today, the many unusual properties of EM were discovered, described, & quantified up until 1861. Then, James Clerk Maxwell (1831 – 1879) (ref [8]) expressed what had been learned through experimentation into equations which bear his name. Oliver Heaviside (refs [9] & [10]) consolidated these equations into their final form to quantify EM phenomena completely (refs [11] & [12]). Once EM characteristics were fully described, the stage was set for further discoveries. At the dawn of the 20th century, Albert Einstein (1879 – 1955) (ref [13]) proposed Special Relativity that explained many EM questions & Max Planck (1858 – 1947) (ref [14]) published his Quantum assumptions which set a path for the frontiers of Physics in the 21st century (ref [15]).

Volts, Amps & Watts Explained: Before we dive into higher level Electromagnetic Theory, let's get some fundamentals clarified. Volts, Amps & Watts are chosen, in part, to be linked through the equation (ref [16]):

$$\textit{Watts} = \textit{Volts} \times \textit{Amps} \quad \Rightarrow \quad \left(P_{Watts}\,\frac{energy}{time}\right) = \left(V_{Volts}\,\frac{energy}{charge}\right) \times \left(I_{Amps}\,\frac{charge}{time}\right)$$

These metric EM units we already use are **one more reason** to go metric! For the following explanation, one must realize we appear to live in an electrically neutral Universe. Every negative electron is paired with an oppositely charged positive proton everywhere we look (ref [17]). If we think of electron-proton pairs as springs, power plants burn MegaTons from King Coal to pull individual negative electrons from positive nuclei of atoms. In so doing, the electron has excess "high quality" energy.

The energy per given number of electrons is stated in Volts. Volts measure the energy per charge. Amps measure the number of energized electrons flowing past a given point in a wire. From the water pipe analogy, Volts represent water pressure, Amps represent water flow (refs [16] & [18]). Eventually, an energized indistinguishable electron pairs up with an atomic nucleus some where @ your place or mine. Then, we burn more coal & repeat!

Direct current (DC) is used in low voltage situations. In a DC circuit, the supplied voltage (V) is constant; the current (I) varies depending on power expended. The Voltage × Ampere multiplication is instantaneous; no time averaging is required (ref [18]).

Alternating current (AC) is used for reason discussed below. As a result, voltages & currents surge back & forth at about 60 times a second. To get the average power dissipated in an application, an averaging factor termed the “root mean square” (RMS – reciprocal square root of two or about 0.7071...) is multiplied by peak voltage measurements (ref [19]).

Maxwell Equation Topics

The ***E*** & ***B*** fields of Maxwell’s Equations denote 3D electric & magnetic vector fields respectively defined for a spatial location @ a particular time in a particular environment. These 3D field vectors are essentially 3 separate component functions specifying a unique direction & magnitude @ each applicable location of interest. The fields are derived from existing scalar EM charges in a given locale along with the vector motion of these charged particles.

Lorentz Force Law: A scalar charge of a particle is a single number describing a property constant of the particle. An introduced charge moving through the defined fields experiences a force as given by Lorentz Force Law (ref [20]):

$$\boldsymbol{F}_t(\boldsymbol{r}_t) = q_t[\boldsymbol{E}(\boldsymbol{r}_t) + \boldsymbol{v}_t \times \boldsymbol{B}(\boldsymbol{r}_t)]$$

This law calculates an EM force ($\boldsymbol{F}_t$) on a particle @ location ($\boldsymbol{r}_t$) with a charge (q_t) traveling with velocity ($\boldsymbol{v}_t$) experiencing an electric field ($\boldsymbol{E}$) & magnetic field ($\boldsymbol{B}$). When Hendrik Lorentz proposed his namesake equation in 1895, magnetism seemed an inexplicable force. However, in 1905, Albert Einstein showed ***B*** fields to be necessary to explain a finite light speed. Furthermore, Maxwell’s equations, in quantifying magnetism, were correct & exact.

Maxwell’s Equations: The four Maxwell’s Equations are heavy into calculus notations that ***cannot*** be described as simple Greek letters. The symbols to be encountered fall in the realm of vector analysis within multi-variable calculus (refs [22] & [23]). However, given a well-defined set of static & moving charge distributions, the four equations give requirements to formulate ***E*** & ***B*** fields to evaluate motion of “test” charges moving within the fields. Without further a due, the four equations are (refs [24] & [25]):

differential form	integral form
1) $\nabla \bullet \boldsymbol{E} = \rho/\varepsilon_0$	$\oiint (\boldsymbol{E} \bullet \hat{\boldsymbol{n}}_s)\, d^2x = \iiint \rho/\varepsilon_0 \, d^3x$
2) $\nabla \bullet \boldsymbol{B} = 0$	$\oiint (\boldsymbol{B} \bullet \hat{\boldsymbol{n}}_s)\, d^2x = 0$
3) $\nabla \, x \, \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0$	$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt}\left(\iint \left(\boldsymbol{B} \cdot \hat{\boldsymbol{n}}_s \right) ds \right)$
4) $\nabla \, x \, \boldsymbol{B} = \left(\mu_0 \boldsymbol{J} + \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t} \right)$	$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 \iint \left(\boldsymbol{J} \cdot \hat{\boldsymbol{n}}_s \right) ds + \frac{1}{c^2} \frac{d}{dt} \left(\iint \left(\boldsymbol{E} \cdot \hat{\boldsymbol{n}}_s \right) ds \right)$

The equations use MKSA (meter / kilogram / second / amp) units including conversion constants, (ε_0 & μ_0), for electric fields from charges & magnetic fields from currents, respectively (ref [21]) in a vacuum. Historical dipole approximations of Maxwell's Equations within dielectric & magnetic media are given here. The four ***new*** symbols (ref [26]) in the above equations are the calculus symbols (∂) ($\int$) (∇▪) (∇**×**).

Calculus Symbols - (∂) ($\int$) **(∇▪) (∇×):** When several independent variables are in a function, the ***partial derivative*** term ($\partial/\partial t$) indicates that the derivative should be performed with the independent denominator time (t), keeping other independent variables (i.e., spatial x, y, z) constant (ref [27]). In Maxwell's Equations, the partial with respect to (wrt) time (t) indicates the last two Maxwell Equations relate ***E*** or ***B*** field changes in time for ***B*** or ***E*** field changes in space, respectively.

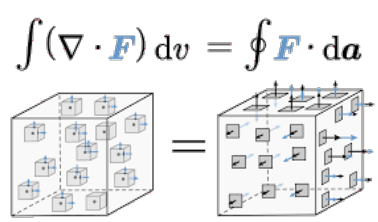
When an ***integration*** ($\int$) is performed, a function is found whose derivative equals the integrand (anti-differentiation). Finding the integral is also a summation of the integrand over the area of interest (ref [28]).

The dot product ***divergence*** (∇▪) term of a vector field performs math calculations that give the slope or gradient of a 3D field in space (ref [29]). This operation results in a scalar that denotes sources or sinks of spatial change in a field.

The cross product ***curl*** (∇**×**) term of a vector field performs a matrix calculus on a vector field to estimate the location's infinitesimal rotation (ref [30]). This operation results in a vector that denotes the rotation axis & magnitude in a field.

Vector Calculus Identities With Integrals: ($\oint$) ($\oiint$) & ($\iint$) ($\iiint$)

The integral form of Maxwell's equations are, in many cases, the preferred form to make calculations. The integral forms use the following identities (ref [31]).

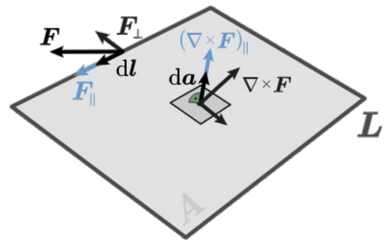


$\iiint (\nabla \,▪\, \boldsymbol{F})\, d^3x = \oiint (\boldsymbol{F} \,▪\, \hat{\boldsymbol{n}}_s)\, d^2x$ (Divergence Theorem)

Divergence Theorem states the dot product divergence **(∇·)** of a vector field (***F***) summed over a volume $(\iiint)_V$ equals the contour integral of the vector field (***F***) & surface normal ($d\boldsymbol{a} = \hat{\boldsymbol{n}}_s da$) dot product over the surface $(\oiint)_s$ enclosing the integral volume (refs

[32] & [33]). In Maxwell's 1st Law, the complete charge configuration within a volume can be gauged by integrating the resultant $\boldsymbol{E}$ field over the surfaces of the enclosed volume.

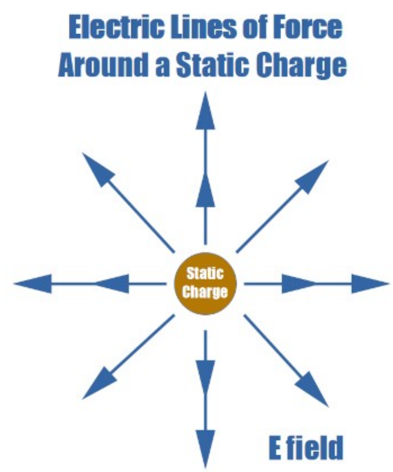
$$\iint [(\nabla \times \boldsymbol{F}) \cdot \hat{\boldsymbol{n}}_s]\, d^2x = \oint (\boldsymbol{F} \cdot d\ell) \quad \text{(Stokes' Theorem)}$$



Stokes' Theorem states the curl **(∇×)** of a vector field (***F***) with the surface normal ($d\boldsymbol{a} = \hat{\boldsymbol{n}}_s d^2x$) dot product summed over an open surface (∬) equals the line integral (∮) of the vector field (***F***) & line tangent ($d\ell$) dot product of a path enclosing the surface (refs [32] & [33]). In Maxwell's 3rd Law, the time rate of change of a $\boldsymbol{B}$ field through a surface is equal to an $\boldsymbol{E}$ field change along the linear perimeter loop around the surface (a measure of Faraday's induction).

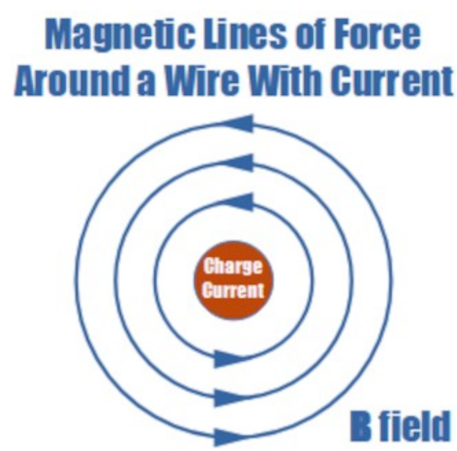
1st Law: Electrostatics

The 1st equation is an expression of Gauss's Law which is directly related to Coulomb's inverse square force law, but for any configuration of charged particles (ref [34]). Gauss's Law incorporates the concept of charge density (ρ) which is charge per unit volume. Time, with the variable (t), does not appear in the equation. The 1st equation relates $\boldsymbol{E}$ field strength & the surrounding charge distributions. Then, it is assumed charges do not move; the equation does not take into account the finite speed of light. If the speed of light were instantaneous, this is the only field equation of the four that would be required. The image to the right shows $\boldsymbol{E}$ field lines which parallel direction of force for a central static charge.



Example: When one puts on a sweater, the person's clothes or their hair on their head stands on end; that's an example of electrostatics. In EM, like charges repel. In a person's electrostatic state with an overabundance of negative electrons on his / her body, each highly mobile electron is trying to get as far away from every other electron as possible.

2nd Law: Magnetostatics



The 2nd equation is an expression of Gauss's Law for Magnetism. When only static $\boldsymbol{B}$ fields are present, only charged particles that are moving experience a right angle force creating somewhat circular trajectory (ref [35]). With Gauss's 1st Law, charges are propelled to / from locations of other charges. In Gauss's 2nd Law, the resultant force on charge particles is @ right angles to charge velocity. From Lorentz Law, if the velocity ($\boldsymbol{v}_t$) of the charge in the magnetic ***B*** field is zero, then no magnetic force is present. The following image shows ***B*** field lines circling a current flow in a central wire. The magnetic force acts at right angles to these lines.

Example: A 12-Volt battery or 12-Volt Direct Current (DC) transformer can be used as a power source. Wrap a single copper wire many times around a steel nail & attach the wire ends to the transformer/battery terminals. Turn on the power & the nail becomes a magnet & will attract steel / iron (ref [36]). When current flows through copper wire @ steady-state (magnetostatic requirement), the ***B*** fields form circles around the wire with the wire @ their center.

3rd Law: Faraday's Law of Induction

The 3rd equation is an expression of Faraday's Law for Induction. The law contains the experimental fact that a time-varying magnetic ***B*** field can maintain an ***E*** field gradient around a loop & a subsequent near static Voltage source for power (ref [37]). The integral form of Maxwell's 3rd Law is more applicable here.

A time-varying magnetic flux is generally contained in a ferrous material. Copper wire is wrapped around the magnetic flux area (n) times to precisely vary the amount of ***B*** field flux that generates a given ***E*** field gradient & Voltage source. In a transformer (ref [38]) per Faraday's Law, current carrying wire is wrapped (n_p) times around a special iron alloy. Elsewhere on the same iron bar or torus, induced current wire will be wrapped (n_s) times. From the Turns Ratio Formula (ref [39]), the ideal voltage step of transformers from "primary" voltage to "secondary" voltage include circuits based on primary (n_p) and secondary (n_s) winding counts is:

$$n_s / V_s = n_p / V_p$$

EM induction is exploited in transformers to transmit electric current over large distances @ extremely high +35k Volts, then stepped down to consumer voltage @ household applications (ref [16]). This is a result of the famous Westinghouse-Edison feud (refs [40] & [41]) & their Alternating Current (AC) vs Direct Current (DC) duel.

Example: If one wants proof of Maxwell's 3rd Law, he / she can look at their monthly electric utility bill. The industry did not care how many elephants were electrocuted (ref [42]) with AC power. US electric utilities could invest efficiently in large power plants isolated from city centers, step up the voltage above 35,000 Volts (ref [43]), transmit the electric power over miles of high power lines, step down the voltage to 120 Volts @ neighborhoods for consumers. This EM application minimized electricity lost as waste heat. In 1896, George Westinghouse demonstrated transmission of electrical power (ref [44]) for 26 miles (42 km) from Niagara Falls to Buffalo, NY (ref [45]).

In wiring his electricity demonstration of New York City, Thomas Edison had to locate DC power plants close to the consumers, could only step down its voltage & used copper wires @ diameters upwards to one inch to carry his DC power (ref [46]). When a capitalistic industry can use an idea to deliver a product @ a lower cost, greed will eventually kick in. Now, dangerous extremely high voltage electrons trickle through high energy power lines to deliver relatively low cost power @ our homes. At times, a lineman has to flip a +35 kV switch to get the power flowing. Then, we have to rely on OSHA & the Feds to regulate the industry!

4th Law: Ampère's Law

The 4th equation is an expression of Ampère's Law relating ***E*** fields, ***B*** fields & electric current density (***J***). This law provides the principal through which electric generators & alternators function. "A magnetic field can be generated by the electric current or changing the electric field" (ref [24]).

The law contains the experimental fact that a time-varying electric ***E*** field can maintain a magnetic ***B*** field gradient around a loop & a subsequent near static magnetic flux with a current flow. Some generators can accept an electric current & power source becoming an electric motor. Then, Ampère's Law can work both ways: a time-varying ***E*** field generates a ***B*** field gradient & current; or an electric current with ***B*** field gradient creates a time-variant ***E*** field.

Magnetostatics: When ***E*** & ***B*** fields are constant over time, magnetostatic information is derived from the 4th Law (ref [25]). The ***B*** field is shown to be related to constant electric current (I) in a thin wire. The integral form of the 4th Law is applicable here:

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \qquad \text{integral form is} \qquad \oint (\boldsymbol{B} \cdot d\ell) = \mu_0 \iint (\boldsymbol{J} \cdot \hat{\boldsymbol{n}}_s)\, d^2x = \mu_0 \mathrm{I}$$

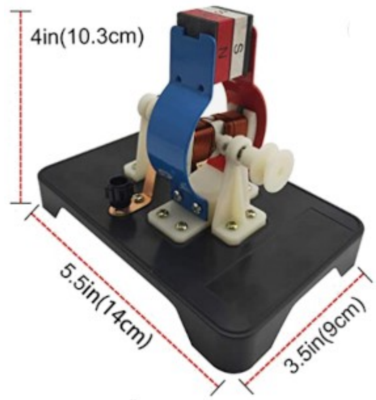
The surface integral over the current density (***J***) vector dot product determines electric current (I) or number of charges passing a single point in a unit interval of time. The 2nd Law indicates that ***B*** fields of equal strength form circles around a static current. Integrating around a circle a radius (r) with the current (I) @ its center:

$$\oint (\boldsymbol{B} \cdot d\ell) = 2 \pi r B = \mu_0 \mathrm{I} \quad \Rightarrow \quad B = (\mu_0 \mathrm{I}) / (2 \pi r)$$

The ***B*** field magnitude is evaluated when the current (I) is concentrated in a thin copper wire. Vector ***B*** fields have a tangential direction along a circle with its center @ the wire of electrical current. See ref [25] for this “right-handed” sign determination of (B & I).

Example: With a multi-meter, permanent magnetic & copper wire, one should be able to generate a detectable current and / or voltage similar to a commercial generator. Form loops with the copper wire, hook the ends of the wire to the multi-meter, then vary the magnetic field within the wire coils with the permanent magnetic.

EUDAX Electric DC Motor Kit: For under $20, a DC motor kit can be purchased (Amazon #B07XWQ35MS). The fully operational DC electric motor operates off of 4 AA batteries. The kit allows the student to assemble & troubleshoot building of the motor, then operate the motor.



DC Electric Motor Kit

The motor implements Maxwell's 2nd / 3rd / 4th laws. Copper windings around rotor armatures create bar magnets. As the [motor rotates](), a changing magnetic field through the armature coils produces an impedance when the rotor tries to align with static magnets on the stator. Once the axis rotates for the rotor field to align with the stator field, brushings & wiring along the rotor divert current to the next armature (ref [16]). This kit presents a simple EM motor. Different designs of electric motors can employ AC power or permanent magnets in the rotor.

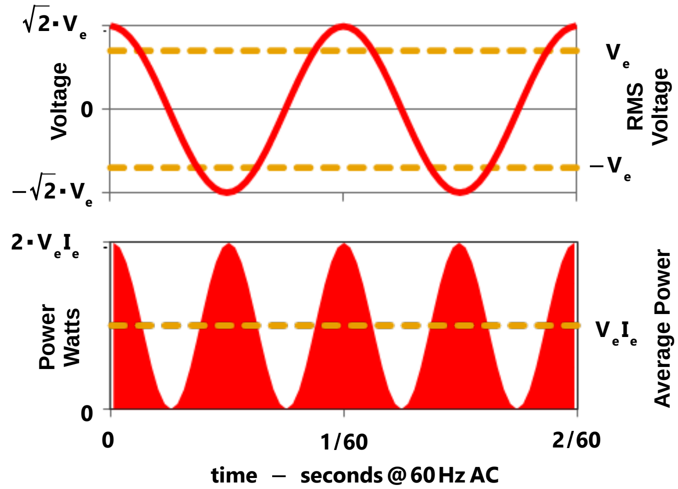
Westinghouse's Power @ Your Electrical Outlet

> **Warning:** Only a licensed electrician should perform modification of electrical wiring in a structure or appliance.

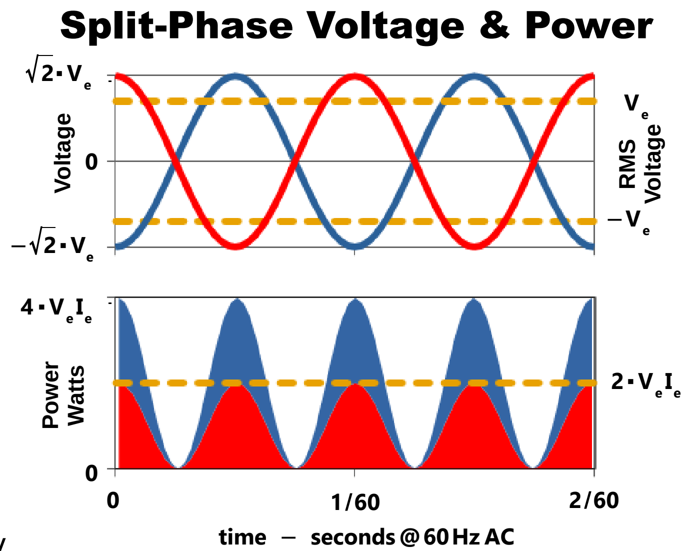
The following plots of Voltage & Power will illustrate the time-dependent behavior of AC electricity @ a typical electrical outlet. In all electrical utility applications, power is supplied @ a constant Voltage. Current load in Amps varies with application. When current exceeds a maximum @ the supplied Voltage source, the circuit is broken by fuse or circuit breaker & supplied Voltage drops to zero (ref [16]).

In the following AC electrical power descriptions, Voltage ($V(t)$) is supplied by the utility company with a peak-to-peak Voltage ($\sqrt{2}\,V_e$) @ a set frequency of sinusoidal cycles per second or Hertz (Hz). An electrical appliance draws a load current with a peak-to-peak Amps ($\sqrt{2}\,I_e$) based on Ohm's Law (refs [47] & [48]) of Resistance ($R \equiv V_e / I_e$) & ($V_e = R\, I_e$). Amplitudes ($V_e = V_{rms}$) & ($I_e = I_{rms}$) are termed root-mean-square (rms) values.

Single-Phase Voltage & Power

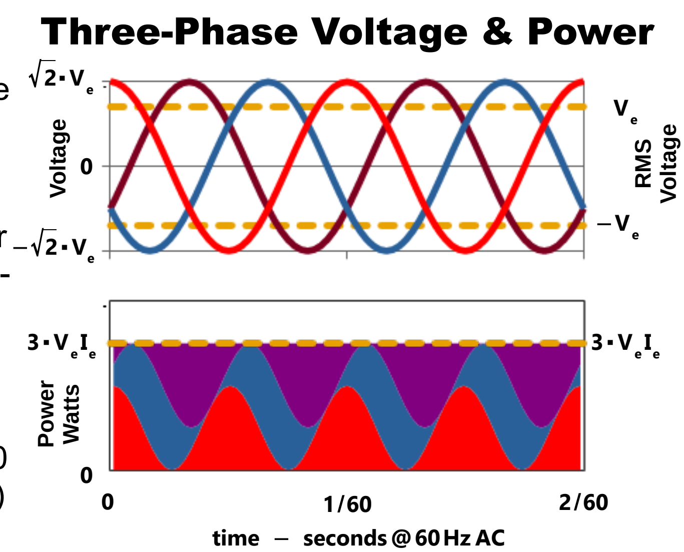


Single-Phase to Ground: The graph above plots a typical 2-wire sinusoidal oscillating Voltage ($V_{red}(t)$) to ground & Power ($P_{red}(t)$) assuming the above form. Standard electricity ($V_e = V_{rms}$) is supplied @ 120 Volts 60 Hz. For England, the standard electricity ($V_e = V_{rms}$) is supplied @ 230 Volts 50 Hz. For this power configuration, a single live wire & a ground wire are needed (ref [49]). The lower plot does indeed show that the rms Voltage (V_e) times rms Current (I_e) equals average Power ($V_e I_e$) generated by the circuit (yellow dashed line).



Split-Phase to Ground: For appliances in the US which draw a heavier load than a single-phase to ground normally supplies, split-phase to ground is available (ref [50]). In this 3-wire configuration, two live wires of standard ($V_e = V_{rms}$) are supplied @ 120 Volts 60 Hz. However, the oscillation is "split" 180º out of phase or half a cycle between the wires.

The graph above plots a split-phase configuration when Voltage ($V(t)$) & Current ($I(t)$) have the described forms. In the US, a pair of singe-wires @ 120 Volts 60 Hz combine to supply electricity @ 240 Volts 60 Hz. Then, a cycle occurs in 1/60 seconds; Voltage ($V_{blue}(t)$) & Power ($P_{blue}(t)$) frequency lags 1/120 seconds from Voltage (V_{red}(t)) & Power (P_{red}(t)) in the plots.



Three-Phase: For most power transmission worldwide, high voltage power lines @ +35k Volts carry 3-phase current (refs [51] & [52]) & invariable require 3 transmission lines @ the same frequency, either 50 Hz or 60 Hz. The reason is shown in the lower power graph of the 3 phases. Continuous cost-efficient electric power is transmitted with no modulation in amplitude when power ($V_e I_e = V_e^2/R$) of all three phases are summed. For 3-phase, each phase is offset in 120º increments with a 1/180 second lag increment ($(V^2_{red}(t) + V^2_{blue}(t) + V^2_{purple}(t))/R = 3V_e I_e = constant$).

Photon Characteristics

Photons are the elements that carry EM radiation in a vacuum @ the speed of light (c) (ref [53]). This fundamental constant has been measured experimentally with a value (si4x6.pdf):

$c = 299792458$ (m/s)

Photon Wavelengths: Wavelengths & frequencies are inherent in the description of photons including the light we see & the microwaves that cook our food. These characteristics are mandated by Maxwell's Equations via the proof below. In a vacuum, no charge is present, then current & charge densities are zero, ($\boldsymbol{J} = 0$ & $\rho = 0$). Maxwell's' Equations for a vacuum are:

1) $\nabla \cdot \boldsymbol{E} = 0$ **3)** $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$

2) $\nabla \cdot \boldsymbol{B} = 0$ **4)** $\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$

For the vector calculus curl operator, the curl of a curl has cancellations, providing the identity on a vector field ($\boldsymbol{F}$) (ref [54]):

$$\nabla \times (\nabla \times \boldsymbol{F}) = \nabla(\nabla \cdot \boldsymbol{F}) - \nabla^2 \boldsymbol{F}$$

Taking the vector curl of Maxwell's Equation (3) & (4):

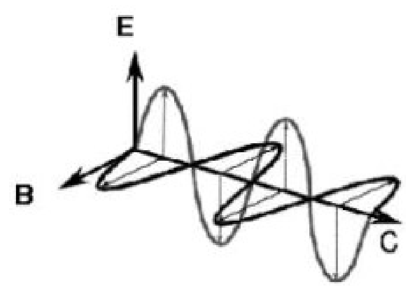
$$\nabla \times \left(\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}\right) = 0 \quad \Rightarrow \quad \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} + \frac{\partial}{\partial t}(\nabla \times \boldsymbol{B}) = 0$$

$$\nabla \times \left(\nabla \times \boldsymbol{B} - \frac{\boldsymbol{1}}{\boldsymbol{c}^2}\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}}\right) = 0 \quad \Rightarrow \quad \nabla(\nabla \cdot \boldsymbol{B}) - \nabla^2 \boldsymbol{B} + \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla \times \boldsymbol{E}) = 0$$

Substituting all of Maxwell's Equations into the above "double curls":

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E} = 0 \quad \& \quad \nabla^2 \boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{B} = 0$$

The above equations are linear 2^{nd} order differential equations that take the form of wave equations (ref [55]) propagating through a medium @ speed (c). Linear in that the coefficients of the derivatives are scalar constants & 2nd-order because the 2^{nd} derivative of a function is required. From higher calculus, the double derivative of ($sin(\varphi)$) is back to ($-sin(\varphi)$) & the double derivative of ($cos(\varphi)$) is similarly ($-cos(\varphi)$).



These trig defined waves solve the above equations & travel through space @ speed (c) with their frequencies taking on any positive value. From the above equations in a vacuum, the ***E*** & ***B*** fields within each photon are also perpendicular. Maxwell's equations mandate a photon's "waves of electricity ... create waves of magnetism, which go on to make waves of electricity and back and forth ... leapfrogging over each other, capable of traveling through space" (refs [56] & [57]).

Quantized Photons: One last thing about photons, they have a quantum energy from Quantum Theory (ref [53]). As most "Quantum" themes of Quantum Theory, the energy of a photon "wave packet" is determined by experiment. In 1905, due to Planck radiation

& photo-electric effect experiments, Albert Einstein proposed that photons should contain a discrete level of energy (E_{photon}) based on their wavelengths (λ_{photon}):

$E_{photon} = hc / \lambda_{photon}$

In the above equation, (h) is Planck Constant, a fundamental quantum constant with a very small value (si4x6.pdf):

$h = 6.62607015 \times 10^{-34}$ (J/Hz)

Visible Photon Energies			
color	wave (nm)	photon energies (J)	(eV)
blue	380	5.23E-19	3.26
yellow	580	3.42E-19	2.14
red	750	2.65E-19	1.65

From the table (ref [58]), individual photons in visible light have discrete wavelength energies (Joules or electron-Volts) (ref [59]) well below thresholds our eyes can detect. Since a Volt is defined as energy (Joules) per charge (Coulomb) & a Coulomb is defined as about $6.242 \times 10^{+18}$ unit electron charges, an electron-Volt (eV) is energy per an electron charge of about $1.602176634 \times 10^{-19}$ Coulombs (si4x6.pdf).

Formal Learning

If the student wants to understand material such as Maxwell's Equations, reading this document as one would read a novel is ***not*** adequate. The information density in a document like this compared to that of English literature differs by orders of magnitude. To comprehend the material contained herein, one may have to re-read the material several times.

If you have half way followed along with all of the Curls, Divergences, Volts, Amps, & Watts, a Science / Technology / Engineering / Mathematics (STEM) profession may be for you. Continue learning by exploring the references below. I received formal EM training from reference [60] (2nd Edition). Freshman year-long Physics & Calculus courses are taught in virtually all US community colleges. Study tips I found useful in my formal STEM education are listed here (eduTips.pdf).

Gaining STEM knowledge takes courage! However, if you are willing to learn STEM material, the educators are willing to teach you. With training in STEM, you ***can*** "leave the world better than you found it."

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